

# HOL-TestGenFW

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March 12, 2010

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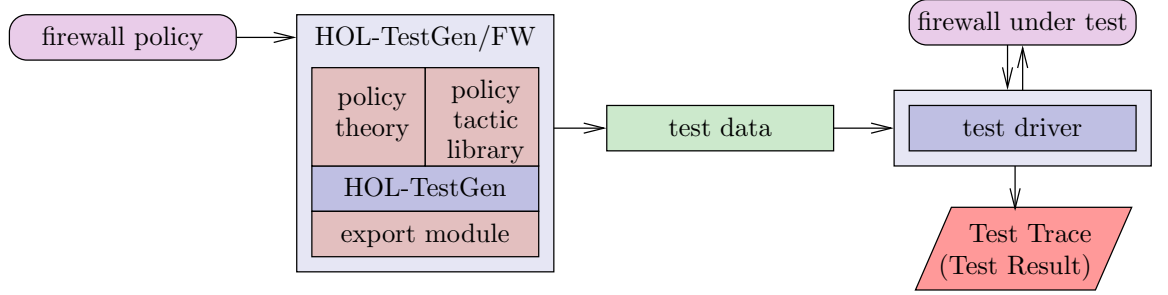


Figure 1: The HOL-TestGen/FW architecture.

## 1 Introduction

As HOL-TestGen is built on the framework of Isabelle with a general plug-in mechanism, HOL-TestGen can be customized to implement domain-specific, model-based test tools in its own right. As an example for such a domain-specific test-tool, we developed HOL-TestGen/FW which extends HOL-TestGen by:

1. a theory (or library) formalizing networks, protocols and firewall policies,
2. domain-specific extensions of the generic test-case procedures (tactics), and
3. support for an export format of test-data for external tools such as [4].

HOL-TestGen/FW is part of the HOL-TestGen distribution. It is located in the directory `examples/hol-testgen-fw`; see [3, 2] for more details.

Figure 1 shows the overall architecture of HOL-TestGen/FW.

In fact, [item 1](#) defines the formal semantics (in HOL) of a specification language for firewall policies; see [2] and the examples provided in the directory `examples/hol-testgen-fw` for details. On the technical level, this library also contains simplification rules together with the corresponding setup of the constraint resolution procedures.

With [item 2](#) we refer to domain-specific processing encapsulated the general HOL-TestGen test-case generation. Since test specifications in our domain have a specific pattern consisting of a limited set of predicates and policy combinators, this can be exploited in specific pre-processing and post-processing of an optimized version of the procedure, now tuned for stateless firewall policies. Moreover, there are new control parameters for the simplification.

With [item 3](#), we refer to an own XML-like format for exchanging test-data for firewalls, i.e., a description of packets to be sent together with the expected behavior of the firewall. This data can be imported in a test-driver for firewalls, for example [4]. This completes our toolchain which, thus,

supports the execution of test data on firewall implementations based on test cases derived from formal specifications.

## 2 Preliminaries

```

theory
  FWTesting
imports
  PacketFilter/PacketFilter
  FWCompilation/FWCompilationProof
  StatefulFW/StatefulFW
  Testing
begin

```

This is a formalisation in Isabelle/HOL of firewall policies and corresponding networks and packets. It first contains the formalisation of stateless packet filters as described in [2], followed by a verified policy normalisation technique (described in [1]), and a formalisation of stateful protocols described in [3].

The following statement adjusts the pre-normalization step of the test case generation algorithm. This turns out to be more efficient for the specific case of firewall policies.

```

setup⟨ map-testgen-params( TestGen.pre-normalizeTNF-tac-update (
  fn ctxt =>
    fn clasimp =>
      ( TestGen.ALLCASES (asm-full-simp-tac (simpset-of
        (ThyInfo.get-theory Int))))))
  ⟩

```

Next, the Isar command *prepare-fw-spec-tac* is specified. It can be used to turn test specifications of the form: " $C\ x \Longrightarrow\ FUT\ x = policy\ x$ " into the desired form for test case generation.

```

ML ⟨
  fun prepare-fw-spec-tac ctxt =
    ( TRY((res-inst-tac ctxt [(x,0),x]) spec 1) THEN
      (resolve-tac [allI] 1) THEN
      (split-all-tac 1) THEN
      (TRY (resolve-tac [impI] 1)))));
  ⟩

```

```

method-setup prepare-fw-spec =
  ⟨

```

```

    Scan.succeed (fn ctxt => SIMPLE-METHOD
      (prepare-fw-spec-tac ctxt)))» Prepares the firewall test theorem
end

```

### 3 Packets and Networks

```

theory NetworkCore
imports Main
begin

```

In networks based e.g. on TCP/IP, a message from A to B is encapsulated in *packets*, which contain the content of the message and routing information. The routing information mainly contains its source and its destination address.

In the case of stateless packet filters, a firewall bases its decision upon this routing information and, in the stateful case, on the content. Thus, we model a packet as a four-tuple of the mentioned elements, together with an id field.

The ID is just an integer:

```

types id = int

```

To enable different representations of addresses (e.g. IPv4 and IPv6, with or without ports), we model them as an unconstrained type class and directly provide several instances:

```

axclass adr < type
types   'α src = 'α::adr
        'α dest = 'α::adr

```

```

instance int ::adr ..
instance nat ::adr ..
instance fun :: (adr,adr) adr ..
instance * :: (adr,adr) adr ..

```

The content is also specified with an unconstrained generic type:

```

types 'β content = 'β

```

For applications where the concrete representation of the content field does not matter (usually the case for stateless packet filters), we provide a default type which can be used in those cases:

```

datatype DummyContent = data

```

A packet is thus:

**types**  $('α, 'β)$  *packet* =  $id \times ('α::adr) \text{ src} \times ('α::adr) \text{ dest} \times 'β \text{ content}$

Please note that protocols (e.g. http) are not modelled explicitly. In the case of stateless packet filters, they are only visible by the destination port, which will be modelled as part of the address. Additionally, stateful firewalls will often determine the protocol by the content of a packet which is thus kept as a generic type.

Port numbers (which are part of an address) are also modelled in a generic way. The integers and the naturals are typical representations of port numbers.

**axclass** *port* < *type*  
**instance** *int* :: *port* ..  
**instance** *nat* :: *port* ..

A packet therefore has two parameters, the first being the address, the second the content. These must of course be specified before we can generate concrete test data later. For the sake of simplicity, we do not allow to have a different address representation format for the source and the destination of a packet respectively.

In order to access the different parts of a packet directly, we define a couple of projectors:

**definition** *id* ::  $('α, 'β)$  *packet*  $\Rightarrow$  *id*  
**where** *id*  $\equiv$  *fst*

**definition** *src* ::  $('α, 'β)$  *packet*  $\Rightarrow$   $('α::adr) \text{ src}$   
**where** *src*  $\equiv$  *fst o snd*

**definition** *dest* ::  $('α, 'β)$  *packet*  $\Rightarrow$   $('α::adr) \text{ dest}$   
**where** *dest*  $\equiv$  *fst o snd o snd*

**definition** *content* ::  $('α, 'β)$  *packet*  $\Rightarrow$   $'β \text{ content}$   
**where** *content*  $\equiv$  *snd o snd o snd*

The following two constants give the source and destination port number of a packet. Address representations using port numbers need to provide a definition for these types.

**consts** *src-port* ::  $('α, 'β)$  *packet*  $\Rightarrow$   $'γ::port$   
**consts** *dest-port* ::  $('α, 'β)$  *packet*  $\Rightarrow$   $'γ::port$

A subnetwork (or simply a network) is a set of sets of addresses.

**types**  $'α \text{ net} = 'α::adr \text{ set set}$

The relation `in_subnet` ( $\sqsubset$ ) checks if an address is in a specific network. It models a kind of subset relation and is defined as an infix operator.

**definition**

$in\_subnet :: 'a::adr \Rightarrow 'a\ net \Rightarrow bool$  (**infixl**  $\sqsubset 100$ ) **where**  
 $in\_subnet\ a\ S \equiv \exists\ s \in S. a \in s$

The following lemmas will be useful later.

**lemma** *in-subnet*:

$((a), e) \sqsubset \{\{(x1), y\}. P\ x1\ y\} \} = (P\ a\ e)$   
**by** (*simp add: in-subnet-def*)

**lemma** *src-in-subnet*:

$((src(q, ((a), e), r, t)) \sqsubset \{\{(x1), y\}. P\ x1\ y\} \}) = (P\ a\ e)$   
**by** (*simp add: in-subnet-def in-subnet src-def*)

**lemma** *dest-in-subnet*:

$((dest(q, r, ((a), e), t)) \sqsubset \{\{(x1), y\}. P\ x1\ y\} \}) = (P\ a\ e)$   
**by** (*simp add: in-subnet-def in-subnet dest-def*)

Address models should provide a definition for the following constant, returning a network consisting of the input address only.

**consts** *subnet-of* ::  $'a::adr \Rightarrow 'a\ net$

**end**

## 4 Address Representations

**theory**

*NetworkModels*

**imports**

*DatatypeAddress*

*DatatypePort*

*IntegerAddress*

*IntegerPort*

*IPv4*

**begin**

One can think of many different possible address representations. In this distribution, we include 5 different versions:

- *DatatypeAddress*: Three explicitly named addresses, which build up a network consisting of three disjunct subnetworks. I.e., there are no

overlaps and there is no way to distinguish between individual hosts within a network.

- **DatatypePort**: An address is a pair, with the first element being the same as above, and the second being a port number modelled as an Integer<sup>1</sup>.
- **IntegerAddress**: An address in an Integer.
- **IntegerPort**: An address is a pair of an Integer and a port (which is again an Integer).
- **IPv4**: An address is a pair. The first element is a four-tuple of Integers, modelling an IPv4 address, the second element is an Integer denoting the port number.

The respective theories of the networks are relatively small. It suffices to provide the respective types, a couple of lemmas, and - if required - a definition for the source and destination ports of a packet.

**end**

## 4.1 Datatype Addresses

```
theory DatatypeAddress
imports NetworkCore
begin
```

A theory describing a network consisting of three subnetworks. Hosts within a network are not distinguished.

```
datatype DatatypeAddress = dmz-adr | intranet-adr | internet-adr
```

**definition**

```
dmz::DatatypeAddress net where
```

```
dmz ≡ {{dmz-adr}}
```

**definition**

```
intranet::DatatypeAddress net where
```

```
intranet ≡ {{intranet-adr}}
```

**definition**

```
internet::DatatypeAddress net where
```

```
internet ≡ {{internet-adr}}
```

**end**

---

<sup>1</sup>For technical reasons, we always use Integers instead of Naturals. As a consequence, the test specifications have to be adjusted to eliminate negative numbers.

## 4.2 Datatype Addresses with Ports

```
theory DatatypePort
imports NetworkCore
begin
```

A theory describing a network consisting of three subnetworks, including port numbers modelled as Integers. Hosts within a network are not distinguished.

```
datatype DatatypeAddress = dmz-adr | intranet-adr | internet-adr
```

```
types
  port = int
  DatatypePort = (DatatypeAddress × port)
```

```
instance DatatypeAddress :: adr ..
```

```
definition
  dmz::DatatypePort net where
  dmz ≡ { {(a,b). a = dmz-adr} }
```

```
definition
  intranet::DatatypePort net where
  intranet ≡ { {(a,b). a = intranet-adr} }
```

```
definition
  internet::DatatypePort net where
  internet ≡ { {(a,b). a = internet-adr} }
```

```
defs (overloaded)
  src-port-def: src-port (x::(DatatypePort,'β) packet) ≡ (snd o fst o snd) x
  dest-port-def: dest-port (x::(DatatypePort,'β) packet) ≡ (snd o fst o snd o snd) x
  subnet-of-def: subnet-of (x::DatatypePort) ≡ { {(a,b). a = fst x} }
```

```
lemma src-port : src-port ((a,x,d,e)::(DatatypePort,'β) packet) = snd x
by (simp add: src-port-def in-subnet)
```

```
lemma dest-port : dest-port ((a,d,x,e)::(DatatypePort,'β) packet) = snd x
by (simp add: dest-port-def in-subnet)
```

```
lemmas DatatypePortLemmas = src-port dest-port src-port-def dest-port-def
end
```

## 4.3 Integer Addresses



```

theory IntegerAddress
imports NetworkCore
begin

```

A theory where addresses are modelled as Integers.

```

types
  IntegerAddress = int

```

```

end

```

#### 4.4 Integer Addresses with Ports

```

theory IntegerPort
imports NetworkCore
begin

```

A theory describing addresses which are modelled as a pair of Integers - the first being the host address, the second the port number.

```

types
  address = int
  port = int
  IntegerPort = address  $\times$  port

```

```

defs (overloaded)

```

```

  src-port-def: src-port (x::IntegerPort,' $\beta$ ) packet  $\equiv$  (snd o fst o snd) x
  dest-port-def: dest-port (x::IntegerPort,' $\beta$ ) packet  $\equiv$  (snd o fst o snd o snd) x
  subnet-of-def: subnet-of (x::IntegerPort)  $\equiv$   $\{\{(a,b). a = \text{fst } x\}\}$ 

```

```

lemma src-port: src-port (a,x::IntegerPort,d,e) = snd x
  by (simp add: src-port-def in-subnet)

```

```

lemma dest-port: dest-port (a,d,x::IntegerPort,e) = snd x
  by (simp add: dest-port-def in-subnet)

```

```

lemmas IntegerPortLemmas = src-port dest-port src-port-def dest-port-def

```

```

end

```

#### 4.5 IPv4 Addresses

```

theory IPv4
imports NetworkCore
begin

```

A theory describing IPv4 addresses with ports. The host address is a four-tuple of Integers, the port number is a single Integer.

```

types
  ipv4-ip = (int × int × int × int)
  port    = int
  ipv4    = (ipv4-ip × port)

defs (overloaded)
  src-port-def: src-port (x::(ipv4,'b) packet) ≡ (snd o fst o snd) x
defs (overloaded)
  dest-port-def: dest-port (x::(ipv4,'b) packet) ≡ (snd o fst o snd o snd) x
defs (overloaded)
  subnet-of-def: subnet-of (x::ipv4) ≡ { {(a,b). a = fst x} }

```

```

definition subnet-of-ip :: ipv4-ip ⇒ ipv4 net
where subnet-of-ip ip ≡ { {(a,b). (a = ip)} }

```

```

lemma src-port: src-port (a,(x::ipv4),d,e) = snd x
by (simp add: src-port-def in-subnet)

```

```

lemma dest-port: dest-port (a,d,(x::ipv4),e) = snd x
by (simp add: dest-port-def in-subnet)

```

```

lemmas IPv4Lemmas = src-port dest-port src-port-def dest-port-def

```

```

end

```

## 5 Policies

### 5.1 Policy Core

```

theory PolicyCore
imports NetworkCore
begin

```

Next, we define the concept of a policy. From an abstract point of view,

a policy is a partial mapping of packets to decisions. Thus, we model the decision as a datatype.

**datatype**  $'\alpha$  out = accept  $'\alpha$  | deny  $'\alpha$

A policy is seen as a partial mapping from packet to packet out.

**types**  $(' \alpha, ' \beta)$  Policy =  $(' \alpha, ' \beta)$  packet  $\rightarrow ((' \alpha, ' \beta)$  packet) out

When combining several rules, the firewall is supposed to apply the first matching one. In our setting this means the first rule which maps the packet in question to *Some* (packet out). This is exactly what happens when using the map-add operator (*rule1* ++ *rule2*). The only difference is that the rules must be given in reverse order.

The constant *p-accept* is *True* if the policy accepts the packet and *False* otherwise.

**definition**

*p-accept* ::  $(' \alpha, ' \beta)$  packet  $\Rightarrow (' \alpha, ' \beta)$  Policy  $\Rightarrow$  bool **where**  
*p-accept* p policy  $\equiv$  policy p = *Some* (accept p)

**end**

## 5.2 Policy Combinators

**theory** PolicyCombinators

**imports**

PolicyCore

**begin**

In order to ease the specification of a concrete policy, we define some combinators. Using these combinators, the specification of a policy gets very easy, and can be done similarly as in tools like IPTables.

**definition**

*allow-all* ::  $(' \alpha, ' \beta)$  Policy **where**  
*allow-all* p  $\equiv$  *Some* (accept p)

**definition**

*deny-all* ::  $(' \alpha, ' \beta)$  Policy **where**  
*deny-all* p  $\equiv$  *Some* (deny p)

**definition**

*allow-all-from* ::  $(' \alpha :: \text{adr})$  net  $\Rightarrow (' \alpha, ' \beta)$  Policy **where**  
*allow-all-from* src-net  $\equiv$  *allow-all* | ' {pa. src pa  $\sqsubset$  src-net}

**definition**

$\text{deny-all-from} :: ('\alpha::\text{adr}) \text{ net} \Rightarrow (''\alpha, '\beta) \text{ Policy where}$   
 $\text{deny-all-from src-net} \equiv \text{deny-all} \mid '\{pa. \text{src pa} \sqsubset \text{src-net}\}$

**definition**

$\text{allow-all-to} :: (''\alpha::\text{adr}) \text{ net} \Rightarrow (''\alpha, '\beta) \text{ Policy where}$   
 $\text{allow-all-to dest-net} \equiv \text{allow-all} \mid '\{pa. \text{dest pa} \sqsubset \text{dest-net}\}$

**definition**

$\text{deny-all-to} :: (''\alpha::\text{adr}) \text{ net} \Rightarrow (''\alpha, '\beta) \text{ Policy where}$   
 $\text{deny-all-to dest-net} \equiv \text{deny-all} \mid '\{pa. \text{dest pa} \sqsubset \text{dest-net}\}$

**definition**

$\text{allow-all-from-to} :: (''\alpha::\text{adr}) \text{ net} \Rightarrow (''\alpha::\text{adr}) \text{ net} \Rightarrow (''\alpha, '\beta) \text{ Policy where}$   
 $\text{allow-all-from-to src-net dest-net} \equiv \text{allow-all} \mid '\{pa. \text{src pa} \sqsubset \text{src-net} \wedge \text{dest pa} \sqsubset \text{dest-net}\}$

**definition**

$\text{deny-all-from-to} :: (''\alpha::\text{adr}) \text{ net} \Rightarrow (''\alpha::\text{adr}) \text{ net} \Rightarrow (''\alpha, '\beta) \text{ Policy where}$   
 $\text{deny-all-from-to src-net dest-net} \equiv \text{deny-all} \mid '\{pa. \text{src pa} \sqsubset \text{src-net} \wedge \text{dest pa} \sqsubset \text{dest-net}\}$

All these combinators and the default rules are put into one single lemma called *PolicyCombinators* to make life easier when we need to unfold a policy consisting of several rules.

**lemmas** *PolicyCombinators* =

$\text{allow-all-def deny-all-def allow-all-from-def deny-all-from-def}$   
 $\text{allow-all-to-def deny-all-to-def allow-all-from-to-def deny-all-from-to-def}$   
 $\text{map-add-def restrict-map-def}$

**end**

### 5.3 Policy Combinators with Ports

**theory** *PortCombinators*

**imports** *PolicyCombinators*

**begin**

This theory defines policy combinators for those network models which have ports. They are provided in addition to the the ones defined in the PolicyCombinators theory.

This theory requires from the network models a definition for the two following constants:

- $\text{src\_port} :: (''\alpha, '\beta) \text{ packet} \Rightarrow (''\gamma :: \text{port})$

- $\text{dest\_port} :: ('α, 'β)\text{packet} \Rightarrow ('γ :: \text{port})$

**definition**

$\text{allow-all-from-port} :: ('α :: \text{adr}) \text{net} \Rightarrow 'γ :: \text{port} \Rightarrow ('α, 'β) \text{Policy}$  **where**  
 $\text{allow-all-from-port src-net s-port} \equiv \text{allow-all-from src-net} \mid ' \{ \text{pa. src-port pa} = \text{s-port} \}$

**definition**

$\text{deny-all-from-port} :: ('α :: \text{adr}) \text{net} \Rightarrow 'γ :: \text{port} \Rightarrow ('α, 'β) \text{Policy}$  **where**  
 $\text{deny-all-from-port src-net s-port} \equiv \text{deny-all-from src-net} \mid ' \{ \text{pa. src-port pa} = \text{s-port} \}$

**definition**

$\text{allow-all-to-port} :: ('α :: \text{adr}) \text{net} \Rightarrow 'γ :: \text{port} \Rightarrow ('α, 'β) \text{Policy}$  **where**  
 $\text{allow-all-to-port dest-net d-port} \equiv \text{allow-all-to dest-net} \mid ' \{ \text{pa. dest-port pa} = \text{d-port} \}$

**definition**

$\text{deny-all-to-port} :: ('α :: \text{adr}) \text{net} \Rightarrow 'γ :: \text{port} \Rightarrow ('α, 'β) \text{Policy}$  **where**  
 $\text{deny-all-to-port dest-net d-port} \equiv \text{deny-all-to dest-net} \mid ' \{ \text{pa. dest-port pa} = \text{d-port} \}$

**definition**

$\text{allow-all-from-port-to} :: ('α :: \text{adr}) \text{net} \Rightarrow 'γ :: \text{port} \Rightarrow ('α :: \text{adr}) \text{net} \Rightarrow ('α, 'β) \text{Policy}$  **where**  
 $\text{allow-all-from-port-to src-net s-port dest-net} \equiv \text{allow-all-from-to src-net dest-net} \mid ' \{ \text{pa. src-port pa} = \text{s-port} \}$

**definition**

$\text{deny-all-from-port-to} :: ('α :: \text{adr}) \text{net} \Rightarrow 'γ :: \text{port} \Rightarrow ('α :: \text{adr}) \text{net} \Rightarrow ('α, 'β) \text{Policy}$  **where**  
 $\text{deny-all-from-port-to src-net s-port dest-net} \equiv \text{deny-all-from-to src-net dest-net} \mid ' \{ \text{pa. src-port pa} = \text{s-port} \}$

**definition**

$\text{allow-all-from-port-to-port} :: ('α :: \text{adr}) \text{net} \Rightarrow 'γ :: \text{port} \Rightarrow ('α :: \text{adr}) \text{net} \Rightarrow 'γ :: \text{port} \Rightarrow ('α, 'β) \text{Policy}$  **where**  
 $\text{allow-all-from-port-to-port src-net s-port dest-net d-port} \equiv \text{allow-all-from-port-to src-net s-port dest-net} \mid ' \{ \text{pa. dest-port pa} = \text{d-port} \}$

**definition**

$\text{deny-all-from-port-to-port} :: ('α :: \text{adr}) \text{net} \Rightarrow 'γ :: \text{port} \Rightarrow ('α :: \text{adr}) \text{net} \Rightarrow 'γ :: \text{port} \Rightarrow ('α, 'β) \text{Policy}$  **where**  
 $\text{deny-all-from-port-to-port src-net s-port dest-net d-port} \equiv \text{deny-all-from-port-to src-net s-port dest-net} \mid ' \{ \text{pa. dest-port pa} = \text{d-port} \}$

**definition**

$\text{allow-all-from-to-port} :: ('α :: \text{adr}) \text{net} \Rightarrow 'γ :: \text{port} \Rightarrow ('α :: \text{adr}) \text{net} \Rightarrow 'γ :: \text{port} \Rightarrow ('α, 'β) \text{Policy}$  **where**

$allow-all-from-to-port \ src-net \ s-port \ dest-net \ d-port \equiv allow-all-from-to \ src-net \ dest-net \mid '$

$$\{pa. \ src-port \ pa = s-port \wedge \ dest-port \ pa = d-port\}$$

**definition**

$deny-all-from-to-port \ :: (' \alpha :: adr) \ net \Rightarrow ' \gamma :: port \Rightarrow (' \alpha :: adr) \ net \Rightarrow ' \gamma :: port \Rightarrow (' \alpha, ' \beta) \ Policy$  **where**

$deny-all-from-to-port \ src-net \ s-port \ dest-net \ d-port \equiv deny-all-from-to \ src-net \ dest-net \mid '$

$$\{pa. \ src-port \ pa = s-port \wedge \ dest-port \ pa = d-port\}$$

**definition**

$allow-from-port-to \ :: ' \gamma :: port \Rightarrow (' \alpha :: adr) \ net \Rightarrow (' \alpha :: adr) \ net \Rightarrow (' \alpha, ' \beta) \ Policy$  **where**

$allow-from-port-to \ port \ src-net \ dest-net \equiv allow-all \mid '$

$$\{pa. \ src \ pa \sqsubset \ src-net \wedge \ dest \ pa \sqsubset \ dest-net \wedge \ (src-port \ pa = port)\}$$

**definition**

$deny-from-port-to \ :: ' \gamma :: port \Rightarrow (' \alpha :: adr) \ net \Rightarrow (' \alpha :: adr) \ net \Rightarrow (' \alpha, ' \beta) \ Policy$  **where**

$deny-from-port-to \ port \ src-net \ dest-net \equiv deny-all \mid '$

$$\{pa. \ src \ pa \sqsubset \ src-net \wedge \ dest \ pa \sqsubset \ dest-net \wedge \ (src-port \ pa = port)\}$$

**definition**

$allow-from-to-port \ :: ' \gamma :: port \Rightarrow (' \alpha :: adr) \ net \Rightarrow (' \alpha :: adr) \ net \Rightarrow (' \alpha, ' \beta) \ Policy$  **where**

$allow-from-to-port \ port \ src-net \ dest-net \equiv allow-all \mid '$

$$\{pa. \ src \ pa \sqsubset \ src-net \wedge \ dest \ pa \sqsubset \ dest-net \wedge \ (dest-port \ pa = port)\}$$

**definition**

$deny-from-to-port \ :: ' \gamma :: port \Rightarrow (' \alpha :: adr) \ net \Rightarrow (' \alpha :: adr) \ net \Rightarrow (' \alpha, ' \beta) \ Policy$  **where**

$deny-from-to-port \ port \ src-net \ dest-net \equiv deny-all \mid '$

$$\{pa. \ src \ pa \sqsubset \ src-net \wedge \ dest \ pa \sqsubset \ dest-net \wedge \ (dest-port \ pa = port)\}$$

**definition**

$allow-from-ports-to \ :: ' \gamma :: port \ set \Rightarrow (' \alpha :: adr) \ net \Rightarrow (' \alpha :: adr) \ net \Rightarrow (' \alpha, ' \beta) \ Policy$  **where**

$allow-from-ports-to \ ports \ src-net \ dest-net \equiv allow-all \mid '$

$$\{pa. \ src \ pa \sqsubset \ src-net \wedge \ dest \ pa \sqsubset \ dest-net \wedge \ (src-port \ pa \in ports)\}$$

**definition**

$allow-from-to-ports \ :: ' \gamma :: port \ set \Rightarrow (' \alpha :: adr) \ net \Rightarrow (' \alpha :: adr) \ net \Rightarrow (' \alpha, ' \beta) \ Policy$  **where**

$allow-from-to-ports \ ports \ src-net \ dest-net \equiv allow-all \mid '$

$$\{pa. \ src \ pa \sqsubset \ src-net \wedge \ dest \ pa \sqsubset \ dest-net \wedge \ (dest-port \ pa \in ports)\}$$

As before, we put all the rules into one lemma called PortCombinators to

ease writing later.

```
lemmas PortCombinators =  
  allow-all-from-port-def deny-all-from-port-def allow-all-to-port-def  
  deny-all-to-port-def allow-all-from-to-port-def  
  deny-all-from-to-port-def  
  allow-from-ports-to-def allow-from-to-ports-def  
  allow-all-from-port-to-def deny-all-from-port-to-def  
  allow-from-port-to-def allow-from-to-port-def deny-from-to-port-def  
  deny-from-port-to-def  
end
```

## 5.4 Ports

```
theory Ports  
imports Main  
begin
```

This theory can be used if we want to specify the port numbers by names denoting their default integer values. If you want to use them, please add "Ports" to the simplifier before test data generation.

```
definition http::int where http  $\equiv$  80  
lemma http1:  $x \neq 80 \implies x \neq \text{http}$   
by (simp add: http-def)
```

```
lemma http2:  $x \neq 80 \implies \text{http} \neq x$   
by (simp add: http-def)
```

```
definition smtp::int where smtp  $\equiv$  25  
lemma smtp1:  $x \neq 25 \implies x \neq \text{smtp}$   
by (simp add: smtp-def)
```

```
lemma smtp2:  $x \neq 25 \implies \text{smtp} \neq x$   
by (simp add: smtp-def)
```

```
definition ftp::int where ftp  $\equiv$  21  
lemma ftp1:  $x \neq 21 \implies x \neq \text{ftp}$   
by (simp add: ftp-def)
```

```
lemma ftp2:  $x \neq 21 \implies \text{ftp} \neq x$   
by (simp add: ftp-def)
```

And so on for all desired port numbers.

```

lemmas Ports = http1 http2 ftp1 ftp2 smtp1 smtp2

end

```

## 6 Policy Normalisation

```

theory
  FWCompilation
imports
  ../PacketFilter/PacketFilter
  Testing
begin

```

This theory contains all the definitions used for policy normalisation as described in [1]. Policy transformations are functions that map policies to policies. We decided to represent policy transformations as *syntactic rules*; this choice paves the way for expressing the entire normalisation process inside HOL by functions manipulating abstract policy syntax.

### 6.1 Basics

We define a very simple policy language:

```

datatype ('α, 'β) Combinators =
  DenyAll
| DenyAllFromTo 'α 'α
| AllowPortFromTo 'α 'α 'β
| Conc (('α, 'β) Combinators) (('α, 'β) Combinators) (infixr ⊕ 80)

```

And define the semantic interpretation of it. For technical reasons, we fix here the type to policies on IntegerPort addresses. However, we could easily provide definitions for other address types as well, using a generic consts for the type definition and a primrec definition for each desired address model.

```

fun C :: (IntegerPort net, port) Combinators ⇒ (IntegerPort, DummyContent)
Policy
where
  C DenyAll = deny-all
| C (DenyAllFromTo x y) = deny-all-from-to x y
| C (AllowPortFromTo x y p) = allow-from-to-port p x y
| C (x ⊕ y) = C x ++ C y

```



## 6.2 Auxiliary definitions and functions.

This subsection defines several functions which are useful later for the combinators, invariants, and proofs.

```
fun position :: 'α ⇒ 'α list ⇒ nat where
  position a [] = 0
| (position a (x#xs)) = (if a = x then 1 else (Suc (position a xs)))
```

```
fun srcNet where
  srcNet (DenyAllFromTo x y) = x
| srcNet (AllowPortFromTo x y p) = x
```

```
fun destNet where
  destNet (DenyAllFromTo x y) = y
| destNet (AllowPortFromTo x y p) = y
```

```
fun srcnets::(IntegerPort net,port) Combinators ⇒ (IntegerPort net) list where
  srcnets DenyAll = []
| srcnets (DenyAllFromTo x y) = [x]
| srcnets (AllowPortFromTo x y p) = [x]
| (srcnets (x ⊕ y)) = (srcnets x)@(srcnets y)
```

```
fun destnets::(IntegerPort net,port) Combinators ⇒ (IntegerPort net) list where
  destnets DenyAll = []
| destnets (DenyAllFromTo x y) = [y]
| destnets (AllowPortFromTo x y p) = [y]
| (destnets (x ⊕ y)) = (destnets x)@(destnets y)
```

```
fun (sequential) net-list-aux where
  net-list-aux [] = []
| net-list-aux (DenyAll#xs) = net-list-aux xs
| net-list-aux ((DenyAllFromTo x y)#xs) = x#y#(net-list-aux xs)
| net-list-aux ((AllowPortFromTo x y p)#xs) = x#y#(net-list-aux xs)
| net-list-aux ((x⊕y)#xs) = (net-list-aux [x])@(net-list-aux [y])@(net-list-aux xs)
```

```
fun net-list where net-list p = remdups (net-list-aux p)
```

```
definition bothNets where bothNets x = (zip (srcnets x) (destnets x))
```

```
fun (sequential) normBothNets where
  normBothNets ((a,b)#xs) = (if ((b,a) ∈ set xs) ∨ (a,b) ∈ set xs then (normBothNets xs) else (a,b)#(normBothNets xs))
| normBothNets x = x
```

```
fun makeSets where
  makeSets ((a,b)#xs) = ({a,b}#(makeSets xs))
| makeSets [] = []
```

```
fun bothNet where
```

```

bothNet DenyAll = {}
| bothNet (DenyAllFromTo a b) = {a,b}
| bothNet (AllowPortFromTo a b p) = {a,b}

```

*Nets\_List* provides from a list of rules a list where the entries are the appearing sets of source and destination network of each rule.

**definition** *Nets\_List* **where** *Nets\_List*  $x = \text{makeSets } (\text{normBothNets } (\text{bothNets } x))$

```

fun (sequential) first-srcNet where
  first-srcNet (x⊕y) = first-srcNet x
| first-srcNet x = srcNet x

```

```

fun (sequential) first-destNet where
  first-destNet (x⊕y) = first-destNet x
| first-destNet x = destNet x

```

```

fun (sequential) first-bothNet where
  first-bothNet (x⊕y) = first-bothNet x
| first-bothNet x = bothNet x

```

```

fun (sequential) in-list where
  in-list DenyAll l = True
| in-list x l = (bothNet x ∈ set l)

```

```

fun all-in-list where
  all-in-list [] l = True
| all-in-list (x#xs) l = (in-list x l ∧ all-in-list xs l)

```

```

fun (sequential) member where
  member a (x⊕xs) = ((member a x) ∨ (member a xs))
| member a x = (a = x)

```

```

fun noneMT where
  noneMT (x#xs) = (dom (C x) ≠ {} ∧ (noneMT xs))
| noneMT [] = True

```

```

fun notMTpolicy where
  notMTpolicy (x#xs) = (if (dom (C x) = {}) then (notMTpolicy xs) else True)
| notMTpolicy [] = False

```

```

fun sdnets where
  sdnets DenyAll = {}
| sdnets (DenyAllFromTo a b) = {(a,b)}
| sdnets (AllowPortFromTo a b c) = {(a,b)}
| sdnets (a ⊕ b) = sdnets a ∪ sdnets b

```

**definition** *packet-Nets* **where** *packet-Nets*  $x \ a \ b \equiv (\text{src } x \sqsubseteq a \wedge \text{dest } x \sqsubseteq b) \vee (\text{src } x \sqsubseteq b \wedge \text{dest } x \sqsubseteq a)$

```

fun matching-rule-rev where
  matching-rule-rev a (x#xs) = (if a ∈ dom (C x) then (Some x) else (matching-rule-rev a xs))
  | matching-rule-rev a [] = None

```

Provides the first matching rule of a policy given as a list of rules.

```

definition matching-rule where
  matching-rule a x ≡ (matching-rule-rev a (rev x))

```

```

definition subnetsOfAdr where subnetsOfAdr a ≡ {x. a ⊆ x}

```

```

definition fst-set where fst-set s ≡ {a. ∃ b. (a,b) ∈ s}

```

```

definition snd-set where snd-set s ≡ {a. ∃ b. (b,a) ∈ s}

```

```

fun memberP where
  memberP r (x#xs) = (member r x ∨ memberP r xs)
  | memberP r [] = False

```

```

fun firstList where
  firstList (x#xs) = (first-bothNet x)
  | firstList [] = {}

```

### 6.3 Invariants

If there is a DenyAll, it is at the first position

```

fun wellformed-policy1:: ((IntegerPort net, port) Combinators) list ⇒ bool where

```

```

  wellformed-policy1 [] = True
  | wellformed-policy1 (x#xs) = (DenyAll ∉ (set xs))

```

There is a DenyAll at the first position

```

fun wellformed-policy1-strong:: ((IntegerPort net, port) Combinators) list ⇒ bool
where
  wellformed-policy1-strong [] = False
  | wellformed-policy1-strong (x#xs) = (x=DenyAll ∧ (DenyAll ∉ (set xs)))

```

All rules appearing at the left of a DenyAllFromTo, have disjunct domains from it (except DenyAll)

```

fun (sequential) wellformed-policy2 where
  wellformed-policy2 [] = True
  | wellformed-policy2 (DenyAll#xs) = wellformed-policy2 xs
  | wellformed-policy2 (x#xs) = ((∀ c a b. c = DenyAllFromTo a b ∧ c ∈ set xs
    → Map.dom (C x) ∩ Map.dom (C c) = {}) ∧ wellformed-policy2 xs)

```

An allow rule is disjunct with all rules appearing at the right of it. This invariant is not necessary as it is a consequence from others, but facilitates some proofs.

**fun** (*sequential*) *wellformed-policy3* **where**  
*wellformed-policy3* [] = *True*  
| *wellformed-policy3* ((*AllowPortFromTo* *a b p*)#*xs*) = (( $\forall r. r \in \text{set } xs \longrightarrow \text{dom } (C\ r) \cap \text{dom } (C\ (\text{AllowPortFromTo } a\ b\ p)) = \{\}$ )  $\wedge$  *wellformed-policy3* *xs*)  
| *wellformed-policy3* (*x*#*xs*) = *wellformed-policy3* *xs*

All two networks are either disjoint or equal.

**definition** *netsDistinct* **where** *netsDistinct* *a b*  $\equiv \neg (\exists x. x \sqsubset a \wedge x \sqsubset b)$

**definition** *twoNetsDistinct* **where** *twoNetsDistinct* *a b c d*  $\equiv \text{netsDistinct } a\ c \vee \text{netsDistinct } b\ d$

**definition** *allNetsDistinct* **where** *allNetsDistinct* *p*  $\equiv \forall a\ b. (a \neq b \wedge a \in \text{set } (\text{net-list } p)) \wedge b \in \text{set } (\text{net-list } p)) \longrightarrow \text{netsDistinct } a\ b$

**definition** *disjSD-2* **where**  
*disjSD-2* *x y*  $\equiv \forall a\ b\ c\ d. ((a,b) \in \text{sdnets } x \wedge (c,d) \in \text{sdnets } y \longrightarrow (\text{twoNetsDistinct } a\ b\ c\ d \wedge \text{twoNetsDistinct } a\ b\ d\ c))$

The policy is given as a list of single rules.

**fun** *singleCombinators* **where**  
*singleCombinators* [] = *True*  
| *singleCombinators* ((*x*⊕*y*)#*xs*) = *False*  
| *singleCombinators* (*x*#*xs*) = *singleCombinators* *xs*

**definition** *onlyTwoNets* **where**  
*onlyTwoNets* *x*  $\equiv ((\exists a\ b. (\text{sdnets } x = \{(a,b)\})) \vee (\exists a\ b. \text{sdnets } x = \{(a,b), (b,a)\}))$

Each entry of the list contains rules between two networks only.

**fun** *OnlyTwoNets* **where**  
*OnlyTwoNets* (*DenyAll*#*xs*) = *OnlyTwoNets* *xs*  
| *OnlyTwoNets* (*x*#*xs*) = (*onlyTwoNets* *x*  $\wedge$  *OnlyTwoNets* *xs*)  
| *OnlyTwoNets* [] = *True*

**fun** *noDenyAll* **where**  
*noDenyAll* (*x*#*xs*) = (( $\neg \text{member } \text{DenyAll } x$ )  $\wedge$  *noDenyAll* *xs*)  
| *noDenyAll* [] = *True*

**fun** *noDenyAll1* **where**  
*noDenyAll1* (*DenyAll*#*xs*) = *noDenyAll* *xs*  
| *noDenyAll1* *xs* = *noDenyAll* *xs*

**fun** *separated* **where**  
*separated* (*x*#*xs*) = (( $\forall s. s \in \text{set } xs \longrightarrow \text{disjSD-2 } x\ s$ )  $\wedge$  *separated* *xs*)  
| *separated* [] = *True*

**fun** *NetsCollected* **where**  
*NetsCollected* (*x*#*xs*) = (((*first-bothNet* *x*  $\neq$  *firstList* *xs*)  $\longrightarrow (\forall a \in \text{set } xs. \text{first-bothNet } x \neq \text{first-bothNet } a)$ )  $\wedge$  *NetsCollected* (*xs*))

| *NetsCollected* [] = *True*

**fun** *NetsCollected2* **where**  
*NetsCollected2* ( $x \# xs$ ) = ( $xs = [] \vee (first\_bothNet\ x \neq firstList\ xs \wedge NetsCollected2\ xs)$ )  
 | *NetsCollected2* [] = *True*

## 6.4 Transformations

The following two functions transform a policy into a list of single rules and vice-versa.

**fun** *policy2list*::(*IntegerPort* *net*, *port*) *Combinators*  $\Rightarrow$  ((*IntegerPort* *net*, *port*) *Combinators*) *list* **where**  
*policy2list* ( $x \oplus y$ ) = (*concat* [(*policy2list* *x*),(*policy2list* *y*)])  
 | *policy2list* *x* = [*x*]

**fun** *list2policy*::(*IntegerPort* *net*, *port*) *Combinators*) *list*  $\Rightarrow$  ((*IntegerPort* *net*, *port*) *Combinators*) **where**  
*list2policy* ( $x \# []$ ) = *x*  
 | *list2policy* ( $x \# y$ ) =  $x \oplus (list2policy\ y)$

Remove all the rules appearing before a *DenyAll*. There are two alternative versions.

**fun** *removeShadowRules1* **where**  
*removeShadowRules1* ( $x \# xs$ ) = (*if* (*DenyAll*  $\in$  *set xs*) *then* ((*removeShadowRules1* *xs*)) *else*  $x \# xs$ )  
 | *removeShadowRules1* [] = []

**fun** *removeShadowRules1-alternative-rev* **where**  
*removeShadowRules1-alternative-rev* [] = []  
 | *removeShadowRules1-alternative-rev* (*DenyAll*  $\# xs$ ) = [*DenyAll*]  
 | *removeShadowRules1-alternative-rev* [*x*] = [*x*]  
 | *removeShadowRules1-alternative-rev* ( $x \# xs$ ) =  $x \# (removeShadowRules1-alternative-rev\ xs)$

**definition** *removeShadowRules1-alternative* **where** *removeShadowRules1-alternative* *p* = *rev* (*removeShadowRules1-alternative-rev* (*rev p*))

Remove all the rules which allow a port, but are shadowed by a deny between these subnets

**fun** *removeShadowRules2*::  
 ((*IntegerPort* *net*, *port*) *Combinators*) *list*  $\Rightarrow$  ((*IntegerPort* *net*, *port*) *Combinators*) *list*  
**where**  
 (*removeShadowRules2* ((*AllowPortFromTo* *x y p*)  $\# z$ )) =  
 (*if* (((*DenyAllFromTo* *x y*)  $\in$  *set z*)) *then* ((*removeShadowRules2* *z*)) *else*  
 (((*AllowPortFromTo* *x y p*)  $\# (removeShadowRules2\ z)$ )))  
 | *removeShadowRules2* ( $x \# y$ ) =  $x \# (removeShadowRules2\ y)$

| *removeShadowRules2* [] = []

Sorting a policy. We first need to define an ordering on rules. This ordering depends on the *Nets\_List* of a policy.

```

fun smaller :: (IntegerPort net, port) Combinators ⇒
    (IntegerPort net, port) Combinators ⇒
    ((IntegerPort net) set) list ⇒ bool

where
    smaller DenyAll x l = True
| smaller x DenyAll l = False
| smaller x y l =
    ((x = y) ∨
     (if (bothNet x) = (bothNet y) then
      (case y of (DenyAllFromTo a b) ⇒ (x = DenyAllFromTo b a)
      | - ⇒ True)
     else
      (position (bothNet x) l <= position (bothNet y) l)))

```

We use insertion sort for sorting a policy.

```

fun insort where
    insort a [] l = [a]
| insort a (x#xs) l = (if (smaller a x l) then a#x#xs else x#(insort a xs l))

fun sort where
    sort [] l = []
| sort (x#xs) l = insort x (sort xs l) l

```

```

fun sorted where
    sorted [] l ⇔ True
| sorted [x] l ⇔ True
| sorted (x#y#zs) l ⇔ smaller x y l ∧ sorted (y#zs) l

```

*separate* works on a sorted policy: it joins the rules which talk about the traffic between the same two networks.

```

fun separate where
    separate (DenyAll#x) = DenyAll#(separate x)
| separate (x#y#z) = (if (first-bothNet x = first-bothNet y)
    then (separate ((x⊕y)#z))
    else (x#(separate(y#z))))
| separate x = x

```

Insert the DenyAllFromTo rules, such that traffic between two networks can be tested individually

```

fun insertDenies where
    insertDenies (x#xs) = (case x of DenyAll ⇒ (DenyAll#(insertDenies xs))
    | - ⇒ (DenyAllFromTo (first-srcNet x) (first-destNet x) ⊕
    (DenyAllFromTo (first-destNet x) (first-srcNet x)) ⊕
    x)#
    (insertDenies xs))

```

| *insertDenies* [] = []

Remove duplicate rules. This is especially necessary as *insertDenies* might have inserted duplicate rules.

The second function is supposed to work on a list of policies. Only rules which are duplicated within the same policy, are removed.

```
fun removeDuplicates where
  removeDuplicates ( $x \oplus xs$ ) = (if member  $x$   $xs$  then (removeDuplicates  $xs$ ) else
 $x \oplus$ (removeDuplicates  $xs$ ))
| removeDuplicates  $x$  =  $x$ 
```

```
fun removeAllDuplicates where
  removeAllDuplicates ( $x \# xs$ ) = ((removeDuplicates ( $x$ ))#(removeAllDuplicates  $xs$ ))
| removeAllDuplicates  $x$  =  $x$ 
```

Remove rules with an empty domain - they never match any packet.

```
fun removeShadowRules3 where
  removeShadowRules3 ( $x \# xs$ ) = (if (dom ( $C$   $x$ ) = {}) then (removeShadowRules3
 $xs$ ) else ( $x \#$ (removeShadowRules3  $xs$ )))
| removeShadowRules3 [] = []
```

Insert a DenyAll at the beginning of a policy.

```
fun insertDeny where
  insertDeny (DenyAll# $xs$ ) = DenyAll# $xs$ 
| insertDeny  $xs$  = DenyAll# $xs$ 
```

Now do everything:

**definition** *sort'*  $p$   $l \equiv$  *sort*  $l$   $p$

**definition** *normalize'*  $p \equiv$  (*removeAllDuplicates* o *insertDenies* o *separate* o (*sort'* (*Nets-List*  $p$ )) o *removeShadowRules2* o *remdups* o *removeShadowRules3* o *insertDeny* o *removeShadowRules1* o *policy2list*)  $p$

**definition** *normalize*  $p \equiv$  *removeAllDuplicates* (*insertDenies* (*separate* (*sort* (*removeShadowRules2* (*remdups* (*removeShadowRules3* (*insertDeny* (*removeShadowRules1* (*policy2list*  $p$ )))))) ((*Nets-List*  $p$ ))))))

**definition** *normalize-manual-order*  $p$   $l \equiv$  *removeAllDuplicates* (*insertDenies* (*separate* (*sort* (*removeShadowRules2* (*remdups* (*removeShadowRules3* (*insertDeny* (*removeShadowRules1* (*policy2list*  $p$ )))))) (( $l$ ))))))

Of course, *normalize* is equal to *normalize'*, the latter looks nicer though.

**lemma** *normalize* = *normalize'*

**by** (*rule ext*, *simp add: normalize-def normalize'-def sort'-def*)

The following definition helps in creating the test specification for the individual parts of a normalized policy.

**definition** *makeFUT* **where** *makeFUT* *FUT* *p* *x* *n* =  
 (*packet-Nets* *x* (*fst*((*normBothNets* (*bothNets* *p*)))!*n*)) (*snd*((*normBothNets* (*bothNets* *p*)))!*n*))  $\longrightarrow$  *FUT* *x* = *C* ((*normalize* *p*)!(*n*+1)) *x*)

**declare** *C.simps* [*simp del*]

**lemmas** *PLemmas* = *C.simps* *dom-def* *PolicyCombinators.PolicyCombinators*  
*PortCombinators.PortCombinators* *src-def* *dest-def* *in-subnet-def*  
*IntegerPort.src-port-def* *IntegerPort.dest-port-def*

**end**

**theory** *FWCompilationProof*  
**imports** *FWCompilation*  
**begin**

This theory contains the complete proofs of the normalisation procedure. In the generated PDF document, we refrain from showing the complete proofs. Rather only the final theorem is provided.

**lemma** *C-eq-compile-manual*:  
 $\llbracket \text{DenyAll} \in \text{set } (\text{policy2list } p); \text{all-in-list } (\text{policy2list } p) \text{ } l; \text{allNetsDistinct } (\text{policy2list } p) \rrbracket$   
 $\implies$   
 $C (\text{list2policy } (\text{removeAllDuplicates } (\text{insertDenies } (\text{separate } (\text{sort } (\text{removeShadowRules2 } (\text{remdups } (\text{removeShadowRules3 } (\text{insertDeny } (\text{removeShadowRules1 } (\text{policy2list } p))))))$   
 $l)))))) = C \text{ } p$   
**apply** (*subst* *C-eq-RAD[symmetric]*)  
**apply** (*rule* *idNMT*)  
**apply** (*simp* *add: C-eqLemmas-id*)  
**apply** (*rule* *C-eq-Until-InsertDenies*)  
**apply** *simp-all*  
**done**

**end**

## 7 Stateful Firewalls

### 7.1 Basic Constructs



```

theory Stateful
imports ../PacketFilter/PacketFilter Testing
begin

```

The simple system of a stateless packet filter is not enough to model all common real-world scenarios. Some protocols need further actions in order to be secured. A prominent example is the File Transfer Protocol (FTP), which is a popular means to move files across the Internet. It behaves quite differently from most other application layer protocols as it uses a two-way connection establishment which opens a dynamic port. A stateless packet filter would only have the possibility to either always open all the possible dynamic ports or not to allow that protocol at all. Neither of these options is satisfactory. In the first case, all ports above 1024 would have to be opened which introduces a big security hole in the system, in the second case users wouldn't be very happy. A firewall which tracks the state of the TCP connections on a system doesn't help here either, as the opening and closing of the ports takes place on the application layer. Therefore, a firewall needs to have some knowledge of the application protocols being run and track the states of these protocols. We next model this behaviour.

The key point of our model is the idea that a policy remains the same as before: a mapping from packet to packet out. We still specify for every packet, based on its source and destination address, the expected action. The only thing that changes now is that this mapping is allowed to change over time. This indicates that our test data will not consist of single packets but rather of sequences thereof.

At first we hence need a state. It is a tuple from some memory to be refined later and the current policy.

```

types (' $\alpha$ , ' $\beta$ , ' $\gamma$ ) FWState = ' $\alpha$   $\times$  (' $\beta$ , ' $\gamma$ ) Policy

```

Having a state, we need of course some state transitions. Such a transition can happen every time a new packet arrives. State transitions can be modelled using a state-exception monad.

```

types (' $\alpha$ , ' $\beta$ , ' $\gamma$ ) FWStateTransition = (' $\beta$ , ' $\gamma$ ) packet  $\Rightarrow$  (unit, (' $\alpha$ , ' $\beta$ , ' $\gamma$ ) FWState)
MON-SE

```

The memory could be modelled as a list of accepted packets.

```

types (' $\beta$ , ' $\gamma$ ) history = (' $\beta$ , ' $\gamma$ ) packet list

```

The next two constants will help us later in defining the state transitions. The constant *before* is *True* if for all elements which appear before the first element for which *q* holds, *p* must hold.

```

consts before :: (' $\alpha \Rightarrow bool$ )  $\Rightarrow$  (' $\alpha \Rightarrow bool$ )  $\Rightarrow$  ' $\alpha$  list  $\Rightarrow$  bool
primrec

```

```

before p q [] = False
before p q (a # S) = (q a ∨ (p a ∧ (before p q S)))

```

Analogously there is an operator *not-before* which returns *True* if for all elements which appear before the first element for which *q* holds, *p* must not hold.

```

consts not-before :: ('α ⇒ bool) ⇒ ('α ⇒ bool) ⇒ 'α list ⇒ bool
primrec
  not-before p q [] = False
  not-before p q (a # S) = (q a ∨ (¬ (p a) ∧ (not-before p q S)))

```

The next two operators can be used to combine state transitions. It takes the first transition which maps to *Some* 'α.

```

definition orelse:: ('α,'β,'γ) FWStateTransition ⇒ ('α,'β,'γ) FWStateTransition
  ⇒ ('α,'β,'γ) FWStateTransition (infixl orelse 100) where
  (f orelse g) x ≡ λ σ. (case f x σ of None ⇒ g x σ | Some y ⇒ Some y)

end

```

## 7.2 FTP Protocol

```

theory FTP
imports
  Stateful
begin

```

The File Transfer Protocol FTP is a well known example of a protocol which uses dynamic ports and is therefore a natural choice to use as an example for our model.

We model only a simplified version of the FTP protocol over IntegerPort addresses, still containing all messages that matter for our purposes. It consists of the following four messages:

1. *ftp-init*: The client contacts the server indicating his wish to get some data.
2. *ftp-port-request p*: The client, usually after having received an acknowledgement of the server, indicates a port number on which he wants to receive the data.
3. *ftp-data*: The server sends the requested data over the new channel. There might be an arbitrary number of such messages, including zero.

4. *ftp-close*: The client closes the connection. The dynamic port gets closed again.

The content field of a packet therefore now consists of either one of those four messages or a default one.

**datatype** *ftp-msg* = *ftp-init*  
                           | *ftp-port-request* *port*  
                           | *ftp-data*  
                           | *ftp-close*  
                           | *other*

We now also make use of the ID field of a packet. It is used as session ID and we make the assumption that they are all unique.

At first, we need some predicates which check if a packet is a specific FTP message and has the correct session ID.

**definition**

*is-init* :: *id*  $\Rightarrow$  (*IntegerPort*, *ftp-msg*) *packet*  $\Rightarrow$  *bool* **where**  
*is-init* *i* *p*  $\equiv$  *id* *p* = *i*  $\wedge$  *content* *p* = *ftp-init*

**definition**

*is-port-request* :: *id*  $\Rightarrow$  *port*  $\Rightarrow$  (*IntegerPort*, *ftp-msg*) *packet*  $\Rightarrow$  *bool* **where**  
*is-port-request* *i* *port* *p*  $\equiv$  *id* *p* = *i*  $\wedge$  *content* *p* = *ftp-port-request* *port*

**definition**

*is-data* :: *id*  $\Rightarrow$  (*IntegerPort*, *ftp-msg*) *packet*  $\Rightarrow$  *bool* **where**  
*is-data* *i* *p*  $\equiv$  *id* *p* = *i*  $\wedge$  *content* *p* = *ftp-data*

**definition**

*is-close* :: *id*  $\Rightarrow$  (*IntegerPort*, *ftp-msg*) *packet*  $\Rightarrow$  *bool* **where**  
*is-close* *i* *p*  $\equiv$  *id* *p* = *i*  $\wedge$  *content* *p* = *ftp-close*

**definition**

*port-open* :: (*IntegerPort*, *ftp-msg*) *history*  $\Rightarrow$  *id*  $\Rightarrow$  *port*  $\Rightarrow$  *bool* **where**  
*port-open* *L* *a* *p*  $\equiv$  *not-before* (*is-close* *a*) (*is-port-request* *a* *p*) *L*

We now have to model the respective state transitions. It is important to note that state transitions themselves allow all packets which are allowed by the policy, not only those which are allowed by the protocol. Their only task is to change the policy. As an alternative, we could have decided that they only allow packets which follow the protocol (e.g. come on the correct ports), but this should in our view rather be reflected in the policy itself.

Of course, not every message changes the policy. In such cases, we do not have to model different cases, one is enough. In our example, only messages 2 and 4 need special transitions. The default says that if the policy accepts the packet, it is added to the history, otherwise it is simply dropped. The policy remains the same in both cases.



following predicate is *True* and is undefined otherwise. We use it to get the number of the port which we want to close. With the if-condition it is assured that such a port exists, but we might have problems if there are several of them. However, due to our assumption that the session IDs are unique, there won't be a problem as long as we do not open several ports in one single protocol run. This should not occur by the definition of the protocol, but if it does, which might happen if we want to test illegal protocol runs, some proof work might be needed.

Now we specify our test scenario in more detail. We could test:

- one correct FTP-Protocol run,
- several runs after another,
- several runs interleaved,
- an illegal protocol run, or
- several illegal protocol runs.

We only do the the simplest case here: one correct protocol run.

There are four different states which are modelled as a datatype.

**datatype** *ftp-states* = *S0* | *S1* | *S2* | *S3*

The following constant is *True* for all sets which are correct FTP runs for a given source and destination address, ID, and data-port number.

**consts**

*is-ftp* :: *ftp-states*  $\Rightarrow$  *IntegerPort*  $\Rightarrow$  *IntegerPort*  $\Rightarrow$  *id*  $\Rightarrow$  *port*  $\Rightarrow$  (*IntegerPort*,*ftp-msg*)

*history*  $\Rightarrow$  *bool*

**primrec**

*is-ftp* *H c s i p* [] = (*H=S3*)

*is-ftp* *H c s i p* (*x#InL*) = ( $\lambda$  (*id*,*sr*,*de*,*co*). (((*id* = *i*  $\wedge$  (  
 $(H=S2 \wedge sr = c \wedge de = s \wedge co = ftp-init \wedge is-ftp S3 c s i p InL) \vee$   
 $(H=S1 \wedge sr = c \wedge de = s \wedge co = ftp-port-request p \wedge is-ftp S2 c s i p$   
 $InL) \vee$   
 $(H=S1 \wedge sr = s \wedge de = (fst c,p) \wedge co = ftp-data \wedge is-ftp S1 c s i p InL) \vee$   
 $(H=S0 \wedge sr = c \wedge de = s \wedge co = ftp-close \wedge is-ftp S1 c s i p InL) )))))$  *x*

This definition is crucial for specifying what we actually want to test. Extending it produces more test cases but increases the time necessary to create them and vice-versa.

The following constant then returns a set of all the historys which denote such a normal behaviour FTP run, again for a given source and destination address, ID, and data-port.

**definition**

*NB-ftp* :: *IntegerPort src*  $\Rightarrow$  *IntegerPort dest*  $\Rightarrow$  *id*  $\Rightarrow$  *port*  $\Rightarrow$  (*IntegerPort,ftp-msg*)  
*history set where*

*NB-ftp s d i p*  $\equiv \{x. (is-ftp S0 s d i p x)\}$

Contrary to the case of a stateless packet filter, a lot of the proof work will only be done during the test *data* generation. This means that we need to add the required lemmas to the simplifier set, such that they will be used. The following additional lemmas are necessary when we use the *IntegerPort* address representation. They should be added to the simplifier set just before test data generation.

**lemma** *subnetOf-lemma*:  $(a::int) \neq (c::int) \implies \forall x \in \text{subnet-of } (a, b::port). (c, d) \notin x$

**apply** (*rule ballI*)

**apply** (*simp add: IntegerPort.subnet-of-def*)

**done**

**lemma** *subnetOf-lemma2*:  $\forall x \in \text{subnet-of } (a::int, b::port). (a, b) \in x$

**apply** (*rule ballI*)

**apply** (*simp add: IntegerPort.subnet-of-def*)

**done**

**lemma** *subnetOf-lemma3*:  $(\exists x. x \in \text{subnet-of } (a::int, b::port))$

**apply** (*rule exI*)

**apply** (*simp add: IntegerPort.subnet-of-def*)

**done**

**lemma** *subnetOf-lemma4*:  $\exists x \in \text{subnet-of } (a::int, b::port). (a, c::port) \in x$

**apply** (*rule bexI*)

**apply** (*simp-all add: IntegerPort.subnet-of-def*)

**done**

**lemma** *port-open-lemma*:  $\neg (Ex (\text{port-open } [] (x::port)))$

**apply** (*simp add: port-open-def*)

**done**

**end**

## 8 Examples

### 8.1 Stateless Example

**theory**

*SimpleDMZIntegerDocument*

**imports**

*FWTesting*  
**begin**

This is a typical example for a small stateless packet filter. There are three subnetworks, with either none or some protocols allowed between them.

We use IntegerPort as the address model.

**constdefs**  
*intranet :: IntegerPort net*  
*intranet*  $\equiv \{\{(a,b) . a = 3\}\}$   
  
*dmz :: IntegerPort net*  
*dmz*  $\equiv \{\{(a,b) . a = 7\}\}$   
  
*internet :: IntegerPort net*  
*internet*  $\equiv \{\{(a,b) . \neg (a=3 \vee a=7)\}\}$

**constdefs**  
*Intranet-DMZ-Port :: (IntegerPort, DummyContent) Policy*  
*Intranet-DMZ-Port*  $\equiv \text{allow-all-from-port-to intranet ftp dmz}$   
  
*Intranet-Internet-Port :: (IntegerPort, DummyContent) Policy*  
*Intranet-Internet-Port*  $\equiv \text{allow-all-from-port-to intranet http internet}$   
  
*Internet-DMZ-Port :: (IntegerPort, DummyContent) Policy*  
*Internet-DMZ-Port*  $\equiv \text{allow-all-from-port-to internet smtp dmz}$

The policy:

**definition** *policy :: (IntegerPort, DummyContent) Policy where*  
*policy*  $\equiv \text{deny-all ++}$   
           *Intranet-Internet-Port ++*  
           *Intranet-DMZ-Port ++*  
           *Internet-DMZ-Port*

**lemmas** *PolicyLemmas = dmz-def internet-def intranet-def*  
           *Intranet-Internet-Port-def Intranet-DMZ-Port-def*  
           *Internet-DMZ-Port-def policy-def*  
           *src-def dest-def in-subnet-def*  
           *IntegerPortLemmas*  
           *content-def*

Only create test cases crossing network boundaries.

**definition** *not-in-same-net :: (IntegerPort, DummyContent) packet  $\Rightarrow$  bool where*  
*not-in-same-net x*  $\equiv (\text{src } x \sqsubset \text{internet} \longrightarrow \neg \text{dest } x \sqsubset \text{internet}) \wedge$   
                    $(\text{src } x \sqsubset \text{intranet} \longrightarrow \neg \text{dest } x \sqsubset \text{intranet}) \wedge$   
                    $(\text{src } x \sqsubset \text{dmz} \longrightarrow \neg \text{dest } x \sqsubset \text{dmz})$

**declare** *Ports* [*simp add*]

The test specification:

```
test-spec not-in-same-net  $x \longrightarrow FUT\ x = policy\ x$   
  apply (prepare-fw-spec)  
  apply (simp add: not-in-same-net-def PolicyLemmas PortCombinators Policy-  
    Combinators)  
  apply (gen-test-cases FUT)  
  apply (simp-all add: PolicyLemmas)  
store-test-thm PolicyTest
```

**testgen-params**[*iterations=100*]

**gen-test-data** *PolicyTest*

The set of generated test data is:

```
FUT ( $-10, (7, http), (-4, -8), data$ ) = Some (deny ( $-10, (7, http), (-4,$   
   $-8), data$ ))  
FUT ( $8, (7, ftp), (1, 0), data$ ) = Some (deny ( $8, (7, ftp), (1, 0), data$ ))  
FUT ( $-7, (7, smtp), (-2, -7), data$ ) = Some (deny ( $-7, (7, smtp), (-2,$   
   $-7), data$ ))  
FUT ( $9, (7, 0), (2, -5), data$ ) = Some (deny ( $9, (7, 0), (2, -5), data$ ))  
FUT ( $-2, (3, http), (7, -1), data$ ) = Some (deny ( $-2, (3, http), (7, -1),$   
   $data$ ))  
FUT ( $-8, (3, ftp), (7, -4), data$ ) = Some (accept ( $-8, (3, ftp), (7, -4),$   
   $data$ ))  
FUT ( $5, (3, smtp), (7, 2), data$ ) = Some (deny ( $5, (3, smtp), (7, 2),$   
   $data$ ))  
FUT ( $-9, (3, 2), (7, 0), data$ ) = Some (deny ( $-9, (3, 2), (7, 0), data$ ))  
FUT ( $-3, (3, http), (6, -9), data$ ) = Some (accept ( $-3, (3, http), (6,$   
   $-9), data$ ))  
FUT ( $2, (3, ftp), (-8, -6), data$ ) = Some (deny ( $2, (3, ftp), (-8, -6),$   
   $data$ ))  
FUT ( $0, (3, smtp), (8, 6), data$ ) = Some (deny ( $0, (3, smtp), (8, 6),$   
   $data$ ))  
FUT ( $-8, (3, -6), (4, -8), data$ ) = Some (deny ( $-8, (3, -6), (4, -8),$   
   $data$ ))  
FUT ( $1, (7, http), (1, -10), data$ ) = Some (deny ( $1, (7, http), (1, -10),$   
   $data$ ))  
FUT ( $2, (7, ftp), (6, 3), data$ ) = Some (deny ( $2, (7, ftp), (6, 3), data$ ))
```



$FUT (1, (7, smtp), (-7, -2), data) = Some (deny (1, (7, smtp), (-7, -2), data))$   
 $FUT (8, (7, 0), (-4, 7), data) = Some (deny (8, (7, 0), (-4, 7), data))$   
 $FUT (-5, (-9, http), (3, 10), data) = Some (deny (-5, (-9, http), (3, 10), data))$   
 $FUT (-10, (-3, ftp), (3, 3), data) = Some (deny (-10, (-3, ftp), (3, 3), data))$   
 $FUT (0, (-9, smtp), (3, -1), data) = Some (deny (0, (-9, smtp), (3, -1), data))$   
 $FUT (-2, (-3, -4), (3, 7), data) = Some (deny (-2, (-3, -4), (3, 7), data))$   
 $FUT (4, (7, http), (3, -1), data) = Some (deny (4, (7, http), (3, -1), data))$   
 $FUT (-3, (7, ftp), (3, -9), data) = Some (deny (-3, (7, ftp), (3, -9), data))$   
 $FUT (-5, (7, smtp), (3, -8), data) = Some (deny (-5, (7, smtp), (3, -8), data))$   
 $FUT (-4, (7, -8), (3, -6), data) = Some (deny (-4, (7, -8), (3, -6), data))$   
 $FUT (-10, (8, http), (3, -3), data) = Some (deny (-10, (8, http), (3, -3), data))$   
 $FUT (-10, (-2, ftp), (3, -9), data) = Some (deny (-10, (-2, ftp), (3, -9), data))$   
 $FUT (3, (6, smtp), (3, 4), data) = Some (deny (3, (6, smtp), (3, 4), data))$   
 $FUT (4, (4, 5), (3, -1), data) = Some (deny (4, (4, 5), (3, -1), data))$   
 $FUT (9, (-6, http), (7, 8), data) = Some (deny (9, (-6, http), (7, 8), data))$   
 $FUT (6, (-10, ftp), (7, -5), data) = Some (deny (6, (-10, ftp), (7, -5), data))$   
 $FUT (-8, (-2, smtp), (7, -4), data) = Some (accept (-8, (-2, smtp), (7, -4), data))$   
 $FUT (-1, (-3, -3), (7, 10), data) = Some (deny (-1, (-3, -3), (7, 10), data))$   
**end**

## 8.2 FTP Example

```

theory FTPTestDocument
imports
  FWTesting
begin

```

In this theory we generate the test data for correct runs of the FTP protocol. As usual, we start with defining the networks and the policy. We use a rather simple policy which allows only FTP connections starting from the intranet going to the internet and denies everything else.

```

constdefs
  intranet :: IntegerPort net
  intranet  $\equiv \{\{(a,e) . a = 3\}\}$ 

  internet :: IntegerPort net
  internet  $\equiv \{\{(a,c) . a > 3\}\}$ 

```

```

constdefs
  ftp-policy :: (IntegerPort,ftp-msg) Policy
  ftp-policy  $\equiv \text{deny-all} ++ \text{allow-from-to-port } (21::\text{port}) \text{ intranet internet}$ 

```

The next two constants check if an address is in the Intranet or in the Internet respectively.

```

constdefs
  is-in-intranet :: IntegerPort  $\Rightarrow$  bool
  is-in-intranet a  $\equiv (\text{fst } a) = 3$ 

  is-in-internet :: IntegerPort  $\Rightarrow$  bool
  is-in-internet a  $\equiv (\text{fst } a) > 3$ 

```

The next definition is our starting state: an empty trace and the just defined policy.

```

constdefs
   $\sigma\text{-}0\text{-ftp} :: (\text{IntegerPort}, \text{ftp-msg}) \text{ history} \times$ 
    (IntegerPort, ftp-msg) Policy
   $\sigma\text{-}0\text{-ftp} \equiv ([], \text{ftp-policy})$ 

```

Next we state the conditions we have on our trace: a normal behaviour FTP run from the intranet to some server in the internet on port 21.

```

constdefs accept-ftp :: (IntegerPort, ftp-msg) history  $\Rightarrow$  bool
  accept-ftp t  $\equiv \exists \ c \ s \ i \ p. \ t \in \text{NB-ftp } c \ s \ i \ p \wedge \text{is-in-intranet } c \wedge \text{is-in-internet}$ 
   $s \wedge (\text{snd } s) = 21$ 

```

```

fun packet-with-id where
  packet-with-id [] i = []
  | packet-with-id (x#xs) i = (if id x = i then (x#(packet-with-id xs i)) else (packet-with-id
    xs i))

```

The depth of the test case generation corresponds to the maximal length of generated traces. 4 is the minimum to get a full FTP protocol run.

**testgen-params** [*depth=4*]

The test specification:

```
test-spec accept-ftp (rev t)  $\longrightarrow$ 
  ( $\sigma$ -0-ftp  $\models$  (os  $\leftarrow$  mbind t FTP-ST; ( $\lambda \sigma$ . Some (FUT (rev t) =  $\sigma$ ,  $\sigma$ ))))
  apply (simp add: accept-ftp-def  $\sigma$ -0-ftp-def)
  apply (rule impI) +
  apply (unfold NB-ftp-def is-in-internet-def is-in-intranet-def)
  apply simp
  apply (gen-test-cases FUT split: HOL.split-if-asm)
  apply (simp-all)
store-test-thm ftp-test
```

We need to add all required lemmas to the simplifier set, such that they can be used during test data generation.

```
lemmas ST-simps = Let-def valid-def unit-SE-def bind-SE-def orelse-def
  in-subnet-def src-def dest-def IntegerPort.dest-port-def
  subnet-of-def id-def port-open-def is-init-def is-data-def
  is-port-request-def is-close-def p-accept-def content-def
  PolicyCombinators PortCombinators is-in-intranet-def
  is-in-internet-def intranet-def internet-def exI subnetOf-lemma
  subnetOf-lemma2 subnetOf-lemma3 subnetOf-lemma4 port-open-lemma
  ftp-policy-def
```

**declare** *ST-simps* [*simp*]

*gen-test-data ftp-test*

**declare** *ST-simps* [*simp del*]

The generated test data look as follows (with the unfolded policy rewritten):

- FUT [(4, (3, 5), (8, 21), *ftp\_close*), (4, (3, 5), (8, 21), *ftp\_port\_request* 4), (4, (3, 5), (8, 21), *ftp\_init*)] = [(4, (3, 5), (8, 21), *ftp\_close*), (4, (3, 5), (8, 21), *ftp\_port\_request* 4), (4, (3, 5), (8, 21), *ftp\_init*), policy]
- FUT [(1, (3, 7), (9, 21), *ftp\_close*), (1, (9, 21), (3, 6), *ftp\_data*), (1, (3, 7), (9, 21), *ftp\_port\_request* 6), (1, (3, 7), (9, 21), *ftp\_init*)] = [(1, (3, 7), (9, 21), *ftp\_close*), (1, (9, 21), (3, 6), *ftp\_data*), (1, (3, 7), (9, 21), *ftp\_port\_request* 6), (1, (3, 7), (9, 21), *ftp\_init*), policy]

**end**

## References

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